Reduced Feedback Techniques for Kerdock based MIMO-OFDM Precoders *

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***Abstract*— Codebook based linear precoders for Multiple Input Multiple Output- Orthogonal Frequency Division Multiplexing (MIMO-OFDM) are used to achieve spatial and frequency diversity by providing partial Channel State Information (CSI) at the transmitter. Recently, a new types of codebooks called Kerdock codebooks were proposed which are more efficient in terms of search complexity and storage than traditional codebooks such as Grassmanian codebooks. In this report various techniques are studied which can be used to reduce the feedback required for kerdock based MIMO-OFDM precoders.**

I. INTRODUCTION

To achieve extremely high data rates MIMO-OFDM precoders are used in all the recent mobile networks such as LTE, 4G, etc. However, using codebook based precoders at the transmitter requires a considerable computation which can be a bottleneck for the small hand held mobile devices. To counter this, [1] introduced a new codebook design, called Kerdock codebooks, which are more efficient in terms of search complexity and storage than previous codebooks such as grassmanian codebooks [2]. In this report various methods that can be used to reduce the feedback requirements for Kerdock based MIMO-OFDM precoders are studied. This can be further used to increase the data rates.

First, various concepts such as Linear Precoding, codebooks, Kerdock codebooks and MIMO-OFDM are explained. After that, various techniques that can be used for reducing the feedback are explored. Finally, the affect of these techniques on the Vector Symbol Error Rate (VSER) for various Signal to Noise Ratios (SNRs) are reported.

Throughout this report following notation is followed : $\mathbf{A}$ denotes a matrix, $\mathbf{a}$ denotes a column vector, $\mathbf{I}_N$ denotes an Identity matrix of dimensions $N \times N$ and $\mathbf{A}^H$ is the complex transpose-conjugate of a matrix $\mathbf{A}$.

II. LIMITED FEEDBACK LINEAR PRECODING

A. MIMO channel

Channel of a MIMO system with N_t transmit antennas and N_r receive antennas can be represented by a matrix $\mathbf{H}$ of dimension $N_r \times N_t$. The input-output relation in such a channel is defined by the following relationship.

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

Here $\mathbf{x}$ is a $N_t \times 1$ transmitted vector, $\mathbf{w}$ is $N_r \times 1$ additive white gaussian noise(AWGN) whose each entry iid distributed according to $CN(0, N_0)$ and $\mathbf{y}$ is a $N_r \times 1$ received vector.

B. Linear Precoding

To utilise the spatial diversity offered by MIMO channel, an input data stream of N_s dimensions, where $1 \leq N_s \leq N_t$, is mapped to a vector of dimension N_t using a precoding matrix. The input data stream complex vector, $\mathbf{s}$, of dimension $N_s \times 1$ is multiplied by the precoding matrix $\mathbf{F}$ of dimension $N_t \times N_s$ to get the transmitted vector $\mathbf{x}$. The precoding Matrix $\mathbf{F}$ is an unitary matrix which implies $\mathbf{F}^H\mathbf{F} = \mathbf{I}_{N_s}$. Therefore, by plugging in the value of $\mathbf{x}$ in the eq. 1 the input-output relation for linear precoded MIMO system is as follows.

$$\mathbf{y} = \mathbf{HFs} + \mathbf{w} \tag{2}$$

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C. Precoding Matrix Selection

In limited feedback linear precoding system, receiver uses its estimate of the channel (channel estimation techniques not discussed in this report) to select a precoder for the receiver.

In one of the most common methods, the receiver chooses a precoder from a fixed set of N possible precoders. This fixed set is called a codebook , which is common to both receiver and transmitter, and is represented by $\mathcal{F} = \{\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_N\}$, where each $\mathbf{F_i}$ is a possible precoder. Receiver than selects a precoder based on some selection criteria and transmits it to the transmitter using $b = \lceil \log_2 N \rceil$ bits.

In this report, for the case of $N_s = 1$ (called beamforming) following selection criteria is used :

$$\hat{\mathbf{f}} = \arg\max_{\mathbf{f}\in\mathcal{F}} \|\mathbf{H}\mathbf{f}\|_2^2 \tag{3}$$

Where $\mathbf{f}$ is the precoder in the beamforming case, $\hat{\mathbf{f}}$ is the selected precoder to be transmitted to the receiver and $\|\cdot\|_2$ is 2-norm of a vector. This criteria was shown to minimize the symbol error probability for maximum ratio combining receiver by [3].

When $1 < N_s \leq N_t$ (called Spatial Multiplexing) the following selection criteria which was proposed by [4] is used :

$$\hat{\mathbf{F}} = \arg\max_{\mathbf{F}\in\mathcal{F}} \lambda_{\min}\{\mathbf{H}\mathbf{F}\} \tag{4}$$

Where $\hat{\mathbf{F}}$ is the selected precoder.

III. Kerdock Codebook

A special type of codebook design was proposed by [1] called Kerdock codebooks. The Kerdock codebook is made up of mutually unbiased bases (MUBs) with quaternary alphabets. It was shown by [1] that these Kerdock codebooks are similar in terms of error rates and more efficient in terms storage & search complexity than previous works such as Grassmannian codebooks [2].

A. Properties

The precoders in Kerdock codebook are quaternary alphabet. This implies that if $\mathbf{F}_i$ is a precoder in $\mathcal{F}$, then all the entries in $\mathbf{F}_i$ belong to the set $\{1, -1, i, -i\}$.

The Kerdock codebooks are also MUBs. This implies for every $N_t \times N_s$ precoders $\mathbf{F}_i = \begin{bmatrix} \mathbf{f}_{i,1} & \cdots & \mathbf{f}_{i,N_s} \end{bmatrix}$ and $\mathbf{F}_j = \begin{bmatrix} \mathbf{f}_{j,1} & \cdots & \mathbf{f}_{j,N_s} \end{bmatrix}$ in the kerdock codebook $\mathcal{F}$ for $i \neq j$, $\mathbf{F}_i \& \mathbf{F}_j$ are orthonormal and $|\langle \mathbf{f}_{i,n}, \mathbf{f}_{j,m}\rangle| = 1/\sqrt{N_t}$ for all $n, m = 1, \ldots, N_t$.

B. Codebook Construction

In the work by [1], two methods for Kerdock codebook construction were proposed namely Sylvester-Hadamard Construction and Power Construction. In this report Power Construction method is used.

Inoue et al [1] utilize the results proven by Gow [5] regarding generation of MUB for constructing kerdock codebooks. Gow proved the following result.

Theorem 1 : Let $\mathbf{D}$ be a $N_t \times N_t$ Unitary Matrix ($\mathbf{D}^H\mathbf{D} = \mathbf{I}_{N_t}$), such that determinant of $\mathbf{D}$ is 1 and $\mathbf{D}^{N_t+1} = \mathbf{I}_{N_t}$. Then set of matrices $\{\mathbf{D}, \mathbf{D}^2, \mathbf{D}^3, \cdots, \mathbf{D}^{N_t}\}$ forms a set of MUB.

The theorem 1 is an existence criteria, the work by [1] finds one such $\mathbf{D}$ for $N_t = 2$ & 4 with quaternary alphabet and uses it to generate the complete codebook using *Theorem 1*. This clearly showcases the storage benefit offered by kerdock codebooks, now the receiver and the transmitter needs to store only one matrix $\mathbf{D}$ instead of the entire codebooks.

It is also interesting to note how can one get a codebook with precoders of dimension $N_t \times N_s$ from the matrix $\mathbf{D}$. Let $\mathcal{S} = \{\mathbf{S}_0, \mathbf{S}_1, \cdots, \mathbf{S}_{N_t}\}$ where $\mathbf{S}_i = \mathbf{D}^{i+1}$, then to construct a N_s stream codebook first a set consisting of all possible combinations of N_s out of N_t columns for each $\mathbf{S}_i$ is constructed. Then N matrices are selected from this set so that the average distance between them is maximum according to some metric as proposed by [3] and [4], to construst the codebook $\mathcal{F}$.

Inoue et al [1] used the following value of $\mathbf{D}$ for $N_t = 4$.

$$\mathbf{D} = \frac{1}{2}\begin{bmatrix} -j & -j & -j & -j \\ 1 & -1 & 1 & -1 \\ -j & -j & j & j \\ -1 & 1 & 1 & -1 \end{bmatrix} \tag{5}$$

IV. MIMO-OFDM

The L tap multi-path structure of MIMO system with N_t transmit and N_r receive antennas can represented by a set of L matrices $\mathcal{H} = \{\mathbf{H}_0, \mathbf{H}_1, \ldots, \mathbf{H}_{L-1}\}$, where each $\mathbf{H}_i$ is a $N_t \times N_r$ matrix. The impulse response of this system is represented by the following equation.

$$\mathbf{H} = \sum_{l=0}^{L-1} \mathbf{H}_l \delta(k-l) \tag{6}$$

Where $\delta(\cdot)$ is the Krocker-Delta function.

In Orthogonal Frequency Division Multiplexing (OFDM) a frequency-selective channel is divided into multiple parallel narrowband flat-fading channels called sub-carriers. Let the number of sub-carriers in which the OFDM divides the frequency-selective channels be N_c, then each of these channels is represented by the following equation.

$$\tilde{\mathbf{H}}_n = \sum_{\ell=0}^{L-1} \mathbf{H}_\ell \exp\left(\frac{-\mathrm{j}2\pi n\ell}{N_\mathrm{c}}\right) \tag{7}$$

Where $\tilde{\mathbf{H}}_n$ for $n = 0, 1, \cdots, N_c - 1$ represents the channel for each sub-carrier.

The input-output in OFDM for the n^{th} sub-carrier is represented by the following equation.

$$\tilde{\mathbf{y}}_n = \tilde{\mathbf{H}}_n \tilde{\mathbf{x}}_n + \tilde{\mathbf{w}}_n \tag{8}$$

Where $\mathbf{x}_n$ is the $N_r \times 1$ input data, $\mathbf{y}_n$ is the $N_t \times 1$ output data and $\mathbf{w}_n$ is the AWGN.

V. Reduced Feedback Techniques

If in a OFDM system the precoder for each of the N_c sub-carriers is calculated at the transmitter then it would require $b \times N_c$ bits for the feedback. However, if precoder is calculated exactly for only a subset of sub-carriers, called pilot sub-carriers, at the receiver and transmitter uses some technique to estimate the precoders for the other sub-carriers than the feedback requirements can be reduced drastically.

All the pilot sub-carriers are separated by a fixed distance of ps called pilot separation. Therefore the set of pilot sub-carriers, represented by $\mathcal{P}$, is given by the following equation.

$$\mathcal{P} = 0 : ps : N_c - 1 \tag{9}$$

Where $\cdot : \cdot : \cdot$ is the standard MATLAB notation. Let K be the number of pilot sub-carriers, then $K = \left\lceil \frac{N_c}{ps} \right\rceil$.

A. Clustering

This is one of the simplest feedback reduction techniques, in which a sub-carrier reuses the precoder for the nearest pilot sub-carrier. For the pilot sub-carriers defined by the eq 9 the nearest pilot sub-carrier for n^{th} sub-carrier where $n = 0, 1, \cdots, N_c - 1$ is given by following equation.

$$\mathbf{nps} = mod(ps \times round(\frac{n}{ps}), N_c) \tag{10}$$

Where $\mathbf{nps}$ stands for nearest pilot sub-carrier, $round(\cdot)$ rounds off its input to the nearest integer and $mod(x, y)$ returns the remainder when x is divided by y. In the eq 10 $mod(\cdot, N_c)$ is used because $\tilde{\mathbf{H}}_{N_c+n} = \tilde{\mathbf{H}}_n$ which can be seen from the eq 7.

The precoders for the pilot sub-carriers in eq 9 are computed exactly using the eqs 3 and 4. Therefore for $n = 0, 1, \cdots, N_c - 1$ the precoder for non-pilot sub-carriers is given by $\mathbf{F}_n = \mathbf{F}_{\mathbf{nps}}$.

B. Geodesic Interpolation

Geodesic is a minimum distance path between two points in a manifold. Work done by [6] uses points on the geodesic between precoder of two pilot sub-carriers to get the precoders for non-pilot sub-carriers for Grassmanian precoders. In this report this work is extended for Kerdock precoders.

For the n^{th} non-pilot sub-carrier first following quantities are calculated.

$$\mathbf{nps}_b = mod(ps \times \left\lfloor \frac{N_c}{ps} \right\rfloor, N_c) \tag{11}$$

$$\mathbf{nps}_a = mod(ps \times \left\lceil \frac{N_c}{ps} \right\rceil, N_c) \tag{12}$$

Where $\mathbf{nps}_b$ and $\mathbf{nps}_a$ stands for nearest pilot sub-carrier below and above respectively. Let m denote the distance of n^{th} sub-carrier from $\mathbf{nps}_b$ and is given by $m = n - \mathbf{nps}_b$. Let precoders corresponding to $\mathbf{nps}_b$ and $\mathbf{nps}_a$ be denoted by $\mathbf{F}_{\mathbf{nps}_b}$ and $\mathbf{F}_{\mathbf{nps}_a}$ respectively.

Next, SVD of $\mathbf{F}^H_{\mathbf{nps}_b}\mathbf{F}_{\mathbf{nps}_a}$ is done and following quantities are calculated.

$$\mathbf{F}^H_{\mathbf{nps}_b}\mathbf{F}_{\mathbf{nps}_a} = \mathbf{USV}^H \tag{13}$$

$$\mathbf{\Sigma} = \frac{K}{N_c}\cos^{-1}(\mathbf{S}) \tag{14}$$

Where $\cos^{-1}(\mathbf{S})$ is a diagonal matrix whose entries are the cos inverse of the diagonal elements of $\mathbf{S}$.

After this, the following quantities are calculated.

$$\overline{\mathbf{F}}_{\mathbf{nps}_b} = \mathbf{F}_{\mathbf{nps}_b}\mathbf{U} \tag{15}$$

$$\overline{\mathbf{F}}_{\mathbf{nps}_a} = \mathbf{F}_{\mathbf{nps}_a}\mathbf{V} \tag{16}$$

$$\mathbf{R} = \left[\overline{\mathbf{F}}_{\mathbf{nps}_b}\mathbf{S} - \overline{\mathbf{F}}_{\mathbf{nps}_a}\right]\left[\sin\left(\frac{N_c}{K}\mathbf{\Sigma}\right)\right]^{-1} \tag{17}$$

Where $\sin(\cdot)$ is a diagonal matrix whose entries the sine values of the input diagnol matrix.

Finally, precoder for the n^{th} non-pilot sub-carrier is calculated is as follows.

$$\mathbf{F}_n = \overline{\mathbf{F}}_{\mathbf{nps}_b}\cos(m\mathbf{\Sigma}) - \mathbf{R}\sin(m\mathbf{\Sigma}) \tag{18}$$

Where $m = n - \mathbf{nps}_b$.

VI. Simulations

In this section Vector Symbol Error Rate (VSER) in various conditions are calculated and reported.

A. Experiment Details

A Rayleigh fading model is used for the MIMO channel with $N_t = N_r = 4$ and whose power delay profile is as follows.

$$\textbf{Powers (in dB)} = [0, -0.9, -4.9, -8, -7.8]$$

$$\textbf{delays} = [0, 2, 8, 12, 23]$$

The $\mathbf{D}$ matrix mentioned in eq 5 is used to construct the codebook. For the spatial multiplexing case the value of N_s is taken as 2. A 64 QAM constellation is used to send and receive the symbols.

A transmitter with perfect Channel State Information (CSI) is used as a baseline model. The precoder for this model is as follows.

$$\mathbf{H} = \mathbf{USV}$$

$$\mathbf{F}_{best\_case} = \mathbf{V}[:, 1:N_s]$$

Where $\mathbf{V}[:, 1:N_s]$ represents first N_s columns of $\mathbf{V}$.

The symbols are decoded at the receiver using following equation.

$$\mathbf{y} = pinv(\mathbf{HF})[\mathbf{HFs} + \mathbf{w}]$$

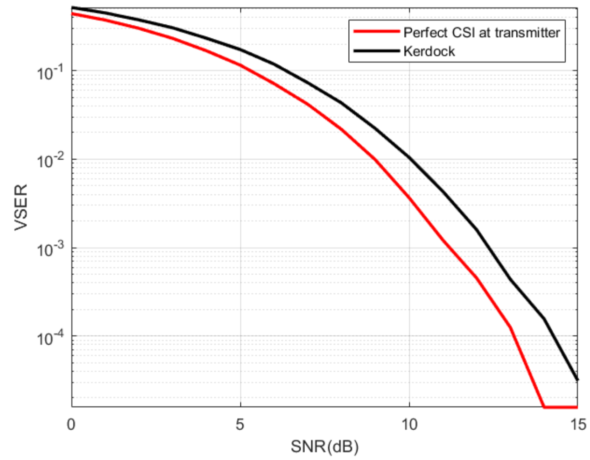

Fig. 1: VSER performance for various SNR of Kerdock codebooks and Perfect CSI at transmitter for beamforming

Where $pinv(\mathbf{A})$ is the Moore-Penrose inverse of a matrix and is given by $pinv(\mathbf{A}) = (\mathbf{A^H A})^{-1}\mathbf{A^H}$.

For MIMO-OFDM the the L-tap multipath MIMO channel is divided in $N_c = 64$ parallel narrowband channels.

B. Using Kerdock Codebook for OFDM-MIMO

In the work done by [1], Kerdock codebooks were used only for flat fading channels. In this report, the kerdock codebooks are extended to L-tap multipath channel where OFDM is used to divide the channel into N_c parallel narrowband sub-carriers. The channel matrix for each of these sub-carriers is first calculated using eq 7 and then precoder for each of these sub-carriers is calculated according to the eq 3 (for beamforming) and eq 4 (for spatial multiplexing). Results for the beamforming and spatial multiplexing are reported in fig 1 and 2 respectively. These results are coherent with ones reported by [1] for the single tap channel.

C. Clustering

The first feedback reduction technique is used for Kerdock codebooks. In this technique instead of calculating exact precoder for each sub-carrier, the precoders are calculated only for a set of pilot sub-carriers which are ps sub-carriers apart from each other. The effect of varying ps on VSER is studied for beamforming and spatial-multiplexing and is shown in figure 3 and 4 respectively.

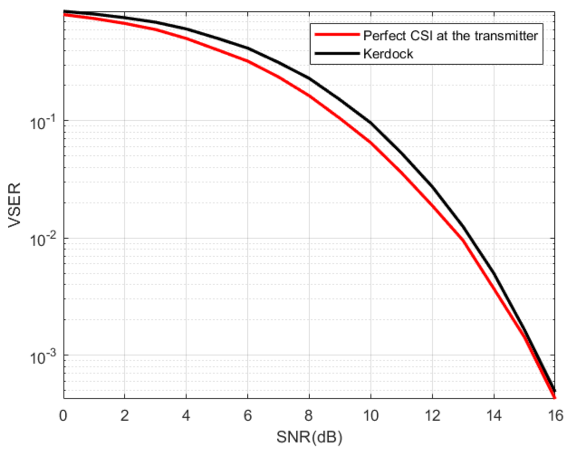

Fig. 2: VSER performance for various SNR of Kerdock codebooks and Perfect CSI at transmitter for $N_s = 2$

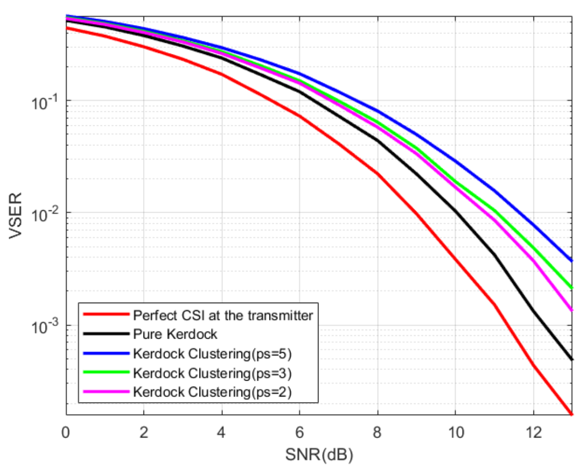

Fig. 3: VSER vs. SNR of clustering techniques ($ps = 2,\ 3\ \&\ 5$), Pure Kerdock and Perfect CSI at the transmitter for beamforming

We expect the graph of clustering to approach the graph when precoder for each sub-carrier is calculated (called Pure Kerdock) as the value of ps decreases, this is also corroborated by the results presented in the figures 3 and 4.

D. Interpolation using Geodesics

Next, interpolation using geodesics for Kerdock precoders is used to reduce the feedback requirements as discussed in the section V-B. The results for $ps = 3$ are reported for the beamforming and spatial-multiplexing cases in the figures 5 and 6 respectively. As expected the VSER graph achieved using this method is in between the pure Kerdock and one achieved using Clustering method as shown in the figure 5 and 6.

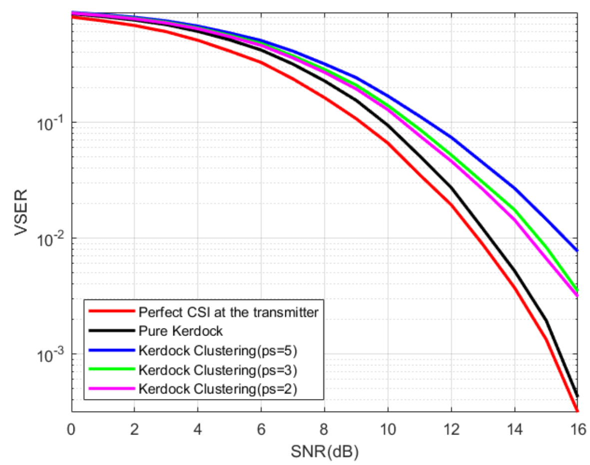

Fig. 4: VSER vs. SNR of clustering techniques ($ps = 2,\ 3\ \&\ 5$), Pure Kerdock and Perfect CSI at the transmitter for $N_s = 2$

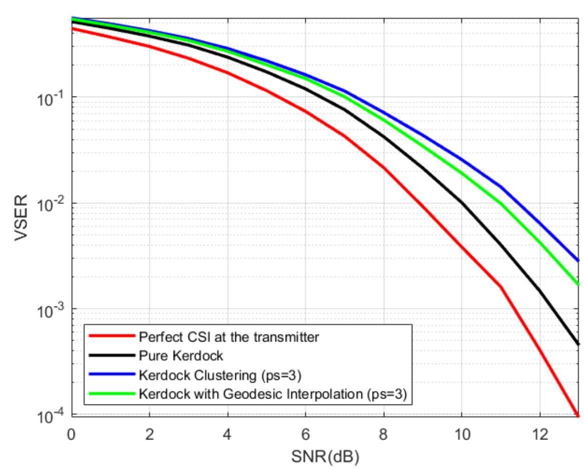

Fig. 5: VSER vs. SNR of interpolation using geodesics ($ps = 3$), clustering techniques ($ps = 3$) and Pure Kerdock for beamforming

VII. Future Work

Currently due to the quaternary alphabets in kerdock codebooks, the input to the $sin(\cdot)$ becomes zero in some cases causing the matrix $\sin\left(\frac{N_c}{K}\mathbf{\Sigma}\right)$ to become singular and as a result its inverse cannot be calculated. Future work to include finding a limiting function that can be used which would prevent this to happen. Also extend the results to bigger values of N_t by finding $\mathbf{D}$ according to the works of [7].

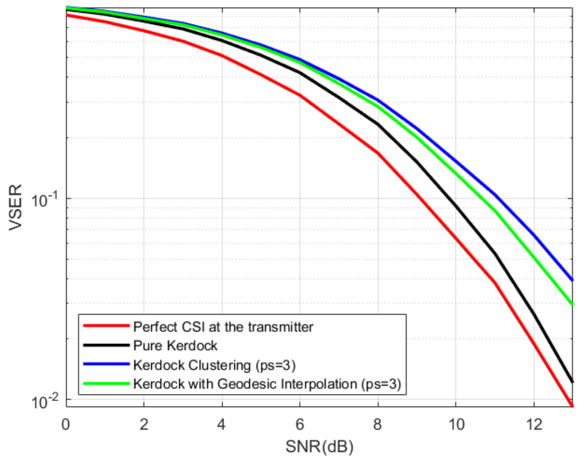

Fig. 6: VSER vs. SNR of interpolation using geodesics ($ps = 3$), clustering techniques ($ps = 3$) and Pure Kerdock for $N_s = 2$

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